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## Smuggling and Parallel Markets for Exports

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SMUGGLING AND PARALLEL MARKETS  
FOR EXPORTS

by

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## ABSTRACT

This paper proposes a model of smuggling consistent with the coexistence of firms involved in strictly legal trade with firms involved in smuggling. A framework is presented in which a firm's degree of risk aversion and the level of government enforcement are the determining factors in the decision of the firm to smuggle or not to smuggle. The model demonstrates that smuggling must be welfare enhancing or all smuggling activity will end.

This paper also provides a theoretical analysis of the effect enforcement has on smuggling and welfare. Increased enforcement is shown to have a negative effect on welfare. Government enforcement is assumed to have two policy instruments it can use to combat smuggling: 1) the probability of detection; 2) the monetary penalty. The relative effectiveness of government enforcement instruments in deterring smuggling is shown to be dependent on the degree of firm risk aversion.

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# SMUGGLING AND PARALLEL MARKETS FOR EXPORTS

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## Introduction

The literature on illegal transactions suggest that smuggling may reduce welfare over the non-smuggling situation. In this essay a model of firm behavior is developed which allows smuggling to be a viable option in addition to legal trade for the domestic firm involved in international trade. The results of the "parallel market" model developed in this essay indicate that if smuggling is not welfare enhancing, regardless of the real resource cost, smuggling will end. In addition, the parallel market model allows the effect of government enforcement on smuggling to be examined. The model demonstrates that increased enforcement has a negative impact on smuggling and, more importantly, a negative impact on welfare.

The seminal paper on illegal transactions by Bhagwati and Hansen (1973) analyzed the welfare implications of smuggling for a small country that imposed a non-prohibitive tariff on imports. Their analysis produced two substantive results. First, under the assumption of perfect competition in the domestic market for imports and the presence of a domestic price differential between legal and illegal goods, illegal trade will dominate the domestic market for imports. In this case a unique welfare ranking is not possible.<sup>1</sup> Second, when it was assumed illegal goods sell at the full duty domestic price, legal and illegal trade coexist in the domestic market. The presence of smuggling, however, has a

negative effect on welfare. Bhagwati and Hansen's results cast doubt on the widely held view that smuggling improves welfare in that it constitutes a partial or total evasion of welfare reducing tariffs.<sup>2</sup>

One weakness of the Bhagwati and Hansen model is its inability to sustain both legal and illegal trade when the domestic price of illegal imports is below the theoretical full duty price for imports. In the real world, legal and illegal trade coexist, with the domestic price of imports being below what the theoretical full duty price should be.

Empirical evidence validating this phenomena is provided by Richard Cooper's (1974) empirical study of import smuggling in Indonesia. Cooper examined the effect of smuggling on the wholesale market price of goods subject to varying tariff levels. Cooper's results revealed that the average wholesale market price of a good subjected to a tariff was only 82% of what the good's tariff inclusive price should be. This result was for goods with a tariff rate of 0% to 100%. For goods that were subject to tariffs of 100% to 200%, only 39% of the tariff increment above 100% was reflected in the average wholesale price. An increase in tariff rates above 258% resulted in a actual reduction in the average wholesale price.<sup>3</sup>

The first theoretical paper focusing on the domestic price effect of smuggling was published by Pitt (1981). Pitt developed a model of export smuggling based on a smuggling production function for firms that use legal export trade as a cover for

their illegal export trade. For the smuggling firm, legal and illegal exports become a joint product of the firm. The average price of the joint export product is greater than the price of exports in the non-smuggling situation. Price disparity is a term Pitt coined for the empirically valid phenomenon of exports selling above their theoretical full duty domestic price.<sup>4</sup>

Pitt's model demonstrates that price disparity and the coexistence of legal and illegal trade are possible. Pitt obtains this result because all firms in his model smuggle and legal and illegal exports are joint products of exporting firms. Pitt's model, however, incorporates passive government enforcement and lacks a decision process to explain of why one firm will smuggle and another will not. These characteristics of Pitt's model produce smuggling's ambiguous welfare effect when smuggling incurs a real resource cost.

Martin and Panagariya (1984) introduced the crime theoretic approach to the analysis of import smuggling. This approach explicitly allows them to incorporate the uncertainty associated with smuggling. As in the Pitt model, however, their analysis fails to provide an explanation of why one firm will smuggle and another will not. The strong assumptions of firm risk neutrality, that all import firms smuggle, and that the smuggling firm sets its own probability of detection generate their ambiguous welfare results.

The three major goals of this essay are: 1) to develop a model that allows varying degrees of firm risk aversion to exist

in the export industry; 2) to provide an analysis of the interaction effect of varying degrees of firm risk aversion with different levels of government enforcement on total smuggling and a country's social welfare; 3) to present a general explanation of the coexistence of firms involved in legal trade only with firms that smuggle when there is a tax wedge driven between the world and domestic price of a good.

For the convenience of the reader an outline of the essay is provided. Section (2) develops the Pitt model of smuggling. A set of modifying assumptions are then introduced which allows the formal development of the parallel market model in the next section. Section (3) formally develops the parallel market model and then analyzes the welfare implications of the interaction effect between the risk associated with smuggling and the existence of parallel markets for exports. Section (4) examines the role of government enforcement policy and its effect on social welfare in the parallel market model. Section (5) gives a summary of the results and discusses the policy implications of those results.

### **Assumptions**

The analysis begins with the Pitt model of smuggling. Pitt's basic model represents the small country case with the terms of trade fixed. The country produces two traded goods, (X) and (M), an exportable and importable, respectively, with primary factors in perfect competition. Production and trade are carried out by identical exporting firms. Legal and illegal trade in



exports is carried out by the same firm. All firms smuggle in the Pitt model, and the law of one price holds in the domestic economy.

It is assumed each firm can trade illegally according to "Pitt's smuggling function",

$$(1) \quad S^* = G(L, S).$$

The term  $(S^*)$  is the quantity of good  $(X)$  smuggled,  $(L)$  is the quantity of good  $(X)$  legally traded and  $(S)$  is the quantity of good  $(X)$  input into smuggling activity. The function  $(G)$  is strictly concave and a twice differentiable linear homogenous function. The function  $(G)$  is also assumed to have the following properties:

$$(2) \quad G_L \geq 0,$$

$$(3) \quad 1 \geq G_S \geq 0,$$

$$(4) \quad S - S^* \geq 0.$$

Assumption (2) states that the marginal product of legal trade used in smuggling is non-negative. Assumption (3) states that a unit increase in the smuggling input  $(S)$  results in a positive, but less than or equal to, unit increase in actual ex-post smuggling. Assumption (4) prohibits the cost of smuggling from being negative. Pitt assumes the difference between ex-ante smuggling  $(S)$  and ex-post smuggling,  $(S^*)$ , consists of penalties and confiscation or a mixture of real resource cost, penalties, and confiscation.

The following assumptions are now made in order to transform the Pitt model into the parallel market model of smuggling: 1)

smuggling by firms may not impose a real cost on society;<sup>5</sup> 2) firms have utility functions with respect to profits that exhibit Von-Neumann-Morgenstern utility properties; 3) smugglers (firms) are natives and, therefore, their utility functions are embodied in the country's social welfare function; 4) export taxes are assumed to be non-prohibitive; 5) all firms in the domestic export industry exhibit diminishing returns in production, which implies (U) shaped average cost curves; 6) firms must bear the risk of illegal activity and they can not insure against criminal penalties; and 7) if the domestic exporting firm decides to smuggle, it will then produce a joint product, and legal trade will act as a cloak for the firm's illegal activity. The firm can use four methods to smuggle exports: a) clandestine smuggling of exports; b) under-invoicing of exports; c) falsely declared exports; d) under-assessment of exports.

I assume that the difference between (S) and (S\*) is a real resource cost coming from either an excess transport cost associated with illegal trade or from evasion tactics used to escape detection.<sup>6</sup> The evasion cost is due to cloaking activities used by smugglers to reduce the expected value of punishment:  $\mu = (\rho \cdot F)$ . The variable ( $\rho$ ) is the probability of detection. The variable (F) is the monetary penalty. Smugglers use cloaking activities to decrease the value of ( $\rho$ ) to less than one. Without cloaking activities, it is assumed ( $\rho$ ) will be equal to one for the smuggling firm. Therefore, it is assumed

the expected value of punishment has a positive relationship with the real resource cost associated with cloaking activities. Hence, the smuggling function  $G(S, L; \mu)$  is assumed to have the following properties with respect to the exogenous variable  $(\mu)$ ,  $G' < 0$ ,  $G'' < 0$ .

The analysis in the next section begins by assuming that smuggling does not incur a real cost over legal trade, i.e.  $(S \approx S^*)$ .<sup>7</sup> This modification assumes that successful ex-post smuggling is less than but approximately equal to ex-ante smuggling to comply with the strict concavity assumption imposed on the smuggling function  $(G)$ . This assumption allows an externalization of the effect enforcement has on smuggling in the Pitt model.

#### **A Model of Smuggling with Parallel Markets for Exports and Risk Aversion**

In all previous articles on illegal trade, except for the one by Scholer (1989), the trading firm had no choice with respect to smuggling. The assumption of parallel markets for exports opens up the possibility of firm choice: strictly legal trade or joint product export smuggling as in the Pitt model. This process will generate two distinct channels through which goods will flow. Then, it is conceivable that the law of one price is no longer valid inside the country. There are several domestic conditions which will promote the development of a parallel market: 1) a domestic market for the export does not exist; 2) all firms, by law must sell their output to the government; and 3) ineffective enforcement of the tax laws.

It is assumed parallel markets for exports and the risk associated with smuggling affect firm behavior. In illegal trade the risk is due to government enforcement of the export tax laws. The firm's decision to become involved in illegal trade, therefore, is based on the potential profits of smuggling and the uncertainty of those profits.

Cooper (1974) alludes to a firm's "threshold of law abidingness," pertaining to a "threshold tariff."<sup>8</sup> I assume that the threshold export tax at which a firm will decide to become involved in illegal trade is determined by the uncertainty associated with illegal profits and the expected value of those profits. An export tax set above this threshold will induce illegal trade, as illegal gains outweigh the economic consequences associated with being caught breaking the law.

It is assumed that each firm has a decision to make: the firm can engage in the Pitt type of smuggling or the firm can sell its output at the legal domestic export tax distorted price ( $P^t$ ),  $P^f \cdot (1-t) = P^t$ . If the firm decides to smuggle, it receives the weighted average price ( $P^s$ ) for its output, equation (8) found below. If the firm decides to stay involved in legal trade, then strictly legal profits for the firm can be represented by the variable ( $Y^l_i$ ),

$$(5) \text{ Max } Y^l_i = P^f \cdot (1-t) \cdot X - C(X, P^l).^9$$

For simplicity let equation (6) represent the firm's profit function for strictly legal export trade,

$$(6) Y^l_i = Y^l_i(P^t, P^l).$$

The variable  $(P^l)$  is the price vector for domestic factor inputs used in the domestic production of exports  $(X)$ , and  $(P^t)$  is the exported goods tax inclusive price or full duty domestic price of the domestically produced exported good  $(X)$ . The function  $C(X, P^l)$  is the firm's cost function, and it is assumed the firm is a price taker in the domestic factor markets. The firm's domestic export production function is assumed to be strictly concave and thus exhibits diminishing returns in production. This restriction on the firm's production function guarantees the firm's average cost curves are (U) shaped.<sup>10</sup>

The firm's expected utility of profit function for legal trade is assumed to be twice differentiable and is denoted  $[E(U(Y_i^l)) = U_i^l]$ . Legal trade is assumed to be a risk free activity.

If the firm decides to become involved in joint product illegal trade, then its situation can be thought of as a lottery. The expected value of the lottery is dependent on the following variables,  $(\rho_i, Y_i^s, F_i)$ . The probability of apprehension  $(\rho)$  is determined by the government. The  $i^{\text{th}}$  firm assumes its probability of being caught is  $(\rho_i)$ . Each firm can influence its probability of success at smuggling by using cloaking activities. The expected value of  $(\rho)$  for the industry, however, is equal to the objective value of  $(\rho)$  set by the government. The variable  $(F_i)$  is the monetary equivalent of the punishment imposed on the  $i^{\text{th}}$  firm by the government if it is caught in the illegal act of smuggling. Fines are considered a transfer to the government.

The variable ( $Y^s_i$ ) represents illegal profits for the individual profit maximizing firm,

$$(7) \text{ Max } Y^s_i = P^f \cdot G(L, S) + P^f \cdot (1-t) \cdot L - C(X, P^l).^{11}$$

Equation (7) represents total profits earned if the firm is successful in illegal trade. Equation (7) implies that legal and illegal trade in exports are considered a joint product of the smuggling firm. The firm's ex-ante output supply price is determined by the weighted average of legal and illegal trade. The smuggler determines his average selling price of output ( $P^s$ ) in order to calculate potential profits coming from successful smuggling,

$$(8) P^s = [P^f \cdot S^* / L+S] + [P^f \cdot (1-t) \cdot L / L+S].$$

For simplicity, let equation (9) represent the firm's profit function for successful smuggling,

$$(9) Y^s_i = Y^s_i(P^s, P^l).$$

By employing Hotelling's lemma it is possible to derive the output supply functions and input demand functions from the profit functions of legal and illegal trade, equations (10) through (13),

$$(10) \partial Y(P^s, P^l) / \partial P^s = S^s(P^s, P^l), S' > 0, S'' > 0, \forall P^s > 0,$$

$$(11) -\partial Y(P^s, P^l) / \partial P^l = x(P^s, P^l), x' < 0, x'' < 0, \forall P^l > 0,$$

$$(12) \partial Y(P^t, P^l) / \partial P^t = L^s(P^t, P^l) L' > 0, L'' > 0, \forall P^t > 0,$$

$$(13) -\partial Y(P^t, P^l) / \partial P^l = x(P^t, P^l) x' < 0, x'' < 0, \forall P^l > 0.$$

The term ( $S^s$ ) is the export supply function for firms that engage in illegal joint product export trade. The term ( $L^s$ ) is the export supply function for firms that do not smuggle exports.

The expected value and variance of profit when the  $i^{\text{th}}$  firm is involved in illegal trade, assuming there is not a real resource cost associated with smuggling, is given in equations (14) and (15),

$$(14) \quad E(Y_i^s) = \rho_i \cdot (Y_i^s - F_i) + (1 - \rho_i) \cdot Y_i^s = Y_i^s - \rho_i \cdot F_i,$$

$$(15) \quad \text{VAR}(Y_i^s) = \rho_i \cdot (1 - \rho_i) \cdot F_i^2.$$

The expected utility function for the firm if it smuggles, is assumed to be twice differentiable and is defined by equation (16),

$$(16) \quad E(U_i^s) = \rho_i \cdot U_i(Y_i^s - F_i) + (1 - \rho_i) \cdot U_i(Y_i^s).$$

Conditions (17) and (18) hold given that the marginal utility of profit is positive as defined by condition (19),

$$(17) \quad \partial E(U_i^s) / \partial \rho_i = U_i(Y_i^s - F_i) - U_i(Y_i^s) < 0,$$

$$(18) \quad \partial E(U_i^s) / \partial F_i = -\rho_i \cdot U'_i(Y_i^s - F_i) < 0,$$

$$(19) \quad \partial E(U_i^s) / \partial Y_i^s = [\rho_i \cdot U'_i(Y_i^s - F_i) + (1 - \rho_i) \cdot U'_i(Y_i^s)] > 0.$$

The  $i^{\text{th}}$  firm bases its decision to become involved in smuggling by comparing expected utilities derived from legal profit with those derived from illegal profit. Therefore, the firm is faced with a random profit ( $Y_i^s$ ) and a certain profit ( $Y_{li}$ ). The firm's decision mechanism is based on a comparison of ( $Y_i^s$ ) and ( $Y_{li}$ ), and this comparison is given in equation (20),

$$(20) \quad E(Y_i^s) - Y_{li} = \pi_i.$$

Applying the theoretical results pertaining to risk and risk aversion developed by Pratt (1964), joint product illegal trade profits represent an uncertain prospect and legal trade profits represent a certain prospect. The term ( $\pi_i$ ) represents the

difference between the expected value of illegal profit and legal profit. It follows that there does exist a  $(\pi_i^* = \pi_i)$  such that the firm is indifferent between legal and illegal trade. The level of  $(\pi_i^*)$  depends on the  $i^{\text{th}}$  firm's attitude toward risk. By applying Pratt's results, it is reasonable to define  $(\pi_i^*)$  as a risk premium. The functional form of  $(\pi^*)$  is given in equation (21),

$$(21) \pi_i^* = (1/2) \cdot \text{VAR}(Y^s_i) \cdot -\{U''(Y^l_i) \div U'(Y^l_i)\}.$$

By employing the standard measure of absolute risk aversion,  $R = -[U''(Y)/U'(Y)]$ , as the measure of the  $i^{\text{th}}$  firm's attitude toward risk, the following conditions arise. The value of  $(\pi_i^*)$  for the  $i^{\text{th}}$  firm will be negative if  $(U'' > 0)$ ; positive if  $(U'' < 0)$ ; and zero if  $(U'' = 0)$ . This implies that the  $i^{\text{th}}$  firm prefers, averts or is neutral towards risk respectively. The variable  $(\pi_i^*)$  represents the insurance premium the firm would be willing to pay if it could insure itself against criminal penalties. Therefore,  $(\pi_i^*)$  represents the minimum level of risk premium necessary to induce the  $i^{\text{th}}$  firm into smuggling. This implies that in equilibrium, at the margin, smugglers that are risk averse earn higher profits than they could in legal activities. Smugglers who are risk preferrers earn lower profits than they could in legal trade. Smugglers who are risk neutral earn the same amount of profits as they could in legal trade. Hence,  $(\pi^*)$  serves as proxy for Cooper's "threshold of law abidingness."<sup>12</sup> Whenever  $(\pi_i > \pi_i^*)$ , the firm will become involved in smuggling if



the firm is assumed to be risk averse or risk neutral as the following condition implies, ( $U'' \leq 0$ ).

It is now assumed that individual attitudes toward risk vary among domestic firms. Let  $(R)$  be a random variable that has a probability density function that generates a finite variance distribution. Attitudes range from risk neutrality ( $R \rightarrow 0$ ) to extreme risk aversion ( $R \rightarrow \infty$ ) with a lower bounded finite variance distribution for  $(R)$ . This assumption implies individual firms displaying risk neutrality or extreme risk aversion are a small proportion of the export industry.

To expand on the concept of industry attitudes towards risk it is assumed that the expected value for the distribution of  $(R)$  is equal to  $(\tau, \tau > 0)$ , as defined by equation (22) below. Equation (22) implies that the industry's average view towards the risk associated with smuggling is risk averse,

$$(22) \quad E(R) = \int_0^{\infty} r \cdot f_r(r) dr = \tau.$$

From the initial assumption of identical firms with varying degrees of risk aversion, the expected value and variance of illegal profits for the representative firm are,

$$(23) \quad E(Y^s) = \rho \cdot (Y^s - F) + (1 - \rho) \cdot Y^s,$$

$$(24) \quad \text{VAR}(Y^s) = \rho \cdot (1 - \rho) \cdot F^2.$$

The decision of an individual firm to continue in only legal trade or to engage in smuggling is based on the comparison of legal profits to the expected value of illegal profits earned by smuggling. This decision process is represented by equations (25) and (26),

$$(25) E(Y^S) - Y^L = \pi,$$

$$(26) \pi^* = (1/2) \cdot \text{VAR}(Y^S) \cdot -\{U''(Y^L) \div U'(Y^L)\}.$$

From equation (26) it is possible to derive  $(\pi^*)$ , the minimum risk premium at which an individual firm in the industry is indifferent to legal or illegal trade. The level of  $(\pi^*)$  depends on the firms attitude towards risk, which is determined by  $(R)$ .

It is assumed that the distribution of the attitudes towards risk for all firms in the industry is given by the probability density function,  $f_r(r)$ . The distribution of  $(R)$  is assumed to have a finite variance distribution for the industry. This implies that  $(\pi^*)$  also has a finite variance distribution for the industry and has a lower bound of zero, thus the minimum risk premium must always be non-negative. The probability density function for  $(\pi^*)$  can be defined as  $f_{\pi^*}(\pi^*)$ .<sup>13</sup>

For example, assume that, on average, the minimum risk premium for the industry is  $(\pi_r)$ . This implies that the expected value of  $(\pi^*)$  is equal to  $(\pi_r)$ , as given by condition (27),

$$(27) E(\pi^*) = \int_0^\infty \pi^* \cdot f_{\pi^*}(\pi^*) d\pi^* = \pi_r.$$

Assume however, that the actual risk premium generated by the export tax,  $(\pi)$ , is some positive value, say  $(\pi_1)$ . Then there will be a set of risk averse firms involved in smuggling. The proportion of the industry involved in smuggling can be determined by the definite integral defined below,

$$(28) \alpha = \int_{\pi^*}^{\pi^1} f_{\pi^*}(\pi^*) d\pi^*.$$

The proportion of the industry involved only in legal trade is, therefore,

$$(29) (1-\alpha) = \int_{\pi^1}^{\pi^0} f_{\pi^*}(\pi^*) d\pi^*.$$

In equation (29), the term  $(1-\alpha)$  represents those firms in the industry for which the level of risk aversion is too high, so they simply do not smuggle. The variable  $(\alpha)$  represents that proportion of the industry involved in smuggling. This implies  $(\alpha)$  represents the proportion of exporting firms selling exports at an average price greater than the selling price of exports for firms that do not smuggle. One minus  $(\alpha)$  represents the proportion of exporting firms selling exports at the theoretical full duty domestic price. Equations (28) and (29) determine the average domestic wholesale price of exports, equation (30).<sup>14</sup> Equation (30) represents a weighted average of trade carried out by the two types of firms in the industry, firms strictly in legal trade and the firms involved in both legal and illegal trade,

$$(30) AWP = \alpha \cdot P^s + (1-\alpha) \cdot p^t. \text{ }^{15}$$

Equations (8) and (30) demonstrate that as the export tax progressively increases, the price of goods exported by the smuggling firm declines by less than the increase in the export tax. This causes  $(\alpha)$  to increase, causing the average domestic wholesale price of exports to decline by a progressively smaller amount than the increase in the export tax. This result implies that the joint product smuggling firm bears only a proportion of

the tax and the legal trade only firm bears the full burden of the tax. This situation forces more firms into smuggling. This is consistent with the empirical results of Cooper for Indonesian imports subject to varying tariff levels.

This section has developed a model of industry behavior which allows firms involved only in legal trade to coexist with firms involved in joint product smuggling. The domestic average wholesale of exports is endogenously determined. These results are obtained because the decision mechanism developed in the model explains why one firm will smuggle and another will not.

The methodology used in this section extended the work of Bhagwati and Hansen, Martin and Panagariya, and Pitt. This extension provides a plausible explanation for the empirically valid case of countries with low tariff or export tax ceilings experiencing little or no smuggling and countries with high tariff or export tax ceilings having a continuing problem with smuggling.

In a recent article on import smuggling, the result of strictly legal traders coexisting with firms that smuggle was demonstrated in a stochastic model of firm behavior developed by Scholer (1989). Scholar assumes varying cost structures between importing firms and varying attitudes toward the risk associated with smuggling by the individual firm's management. These assumptions allow firms in strictly legal trade to coexist in the domestic market with firms that smuggle. The decision to smuggle in Scholer's model is based on the comparison of expected unit

cost of smuggling to the tariff and the firm's attitude toward the risk associated with smuggling. Scholer, however, does not fully develop the Arrow-Pratt methodology that would provide a stronger theoretical foundation for his firm's decision process.

Another weakness of the Scholer model is the way smuggling cost are defined. Scholer assumes the cost of smuggling is the value of expected punishment, i.e., the international value of illegal import confiscations. The real resource cost associated with smuggling in Scholer's model is generated when the government destroys the international value of expected illegal import confiscations. This is a very strong assumption, and Scholer notes that if this assumption is relaxed his results may change.

### **The Welfare Effect of Smuggling**

In the last section it was demonstrated that whenever  $(\pi > \pi^*)$  for any firm or firms, a proportion of the industry will be involved in smuggling. The stage is now set for an examination of how the introduction of smuggling affects total exports, denoted  $(I)$ . The small country assumption made earlier implies that the demand for exports is perfectly elastic. The amount of exports is therefore determined by supply. Assuming that the level of total exports, after the export tax is levied but before smuggling is introduced, is equal to  $X_1^L$ ,

$$(31) \quad I_1 = X_1^L.$$

After smuggling is introduced, the level of total exports is equal to the sum of legal and illegal trade,

$$(32) \quad I_2 = X_2^L + X_2^S.$$

Assuming the supply of total exports has a positive relationship with the price of exports, one would expect that whenever ( $P^{AWP} \geq P^t$ ) then ( $I_2 \geq I_1$ ): ( $P^{AWP}$ ) represents the average wholesale price of exports. Given that ( $X_2^S \geq 0$ ), then  $X_2^L + X_2^S \geq I_1$ , or equivalently,  $X_2^S \geq I_1 - X_2^L$ . Using (31), this implies condition (33),

$$(33) \quad X_2^S \geq X_1^L - X_2^L.$$

Equation (33) demonstrates that the unit volume of illegal trade is greater than the change in unit volume of legal trade. However, as in Pitt's paper, it can not be determined if legal trade increases or decreases because of the change in relative prices; thus, the effect on export tax revenues is ambiguous.<sup>16</sup>

The welfare effect of smuggling with parallel markets and varying degrees of risk aversion will now be examined. The analysis begins from a export tax distorted equilibrium point depicted in figure (1) in appendix (C): production and consumption are equal to ( $P^t, C^t$ ), and welfare is equal to ( $V^t$ ). The term ( $V^t$ ) is an ex-post indirect social welfare function and is defined as  $[V(AWP, Y)]$ .

In the non-smuggling case, the full duty price of exports ( $P^t$ ) is the average wholesale price of exports (AWP). The variable ( $Y$ ) represents income earned by the traded goods industry and government revenues earned from export tax revenues and fines collected.<sup>17</sup> The function ( $V$ ) is assumed to be twice continuously differentiable and non-increasing in (AWP) and non-

decreasing in  $(Y)$ . The introduction of smuggling is assumed to cause a domestic output supply price differential for domestic firms producing exports,  $(P^s > P^t)$ .

Assuming that smugglers do not incur a real resource cost over legal trade, the cost of smuggling is penalties and confiscation. The magnitude of the cost is determined by the fixed level of government enforcement. Therefore the aggregate rate of transformation in trade is the free trade terms of trade. Production and consumption will be equal to  $(P^{awp0})$  and  $(C^{awp0})$  and welfare at  $(V^{awp0})$ . The introduction of smuggling improves welfare over the non-smuggling situation,  $(V^{awp0} > V^t)$  as shown in figure (1).<sup>18</sup>

If it is assumed government enforcement imposes a real cost on smuggling, as it was assumed in the earlier literature, then welfare levels are bounded by  $(V^{awp0})$  and  $(V^t)$ . This can be demonstrated by first assuming the real cost of smuggling is due to real resources being used up in evasion or cloaking activities. Second, assume the expected value of punishment,  $(\mu = \rho \cdot F)$ , is used by smugglers to determine the real resource cost of evasion tactics. As  $(\rho)$  or  $(F)$  increases, the value of  $(S^*)$  declines as  $(G_s)$  declines; thus,  $(P^s)$  declines, as shown in equation (8). Equations (19) and (10) imply expected illegal profits decline and the output from firms involved in illegal trade declines respectively. Therefore, the risk premium earned from smuggling is lower, causing the most risk averse smugglers to end their illegal activity. With fewer firms smuggling, the

average wholesale price of exports will decline and domestic relative prices will deteriorate, welfare will decline as a result of the introduction of real cost, ( $v^{amp0} > v^{amp1}$ ) (see figure 1).

The cost of smuggling is a mix of real resource cost and penalties-confiscation, with legal and illegal firms coexisting in the domestic market.<sup>19</sup> Therefore production and prices are independent of the cost mix but are not independent of the firm mix. Welfare is bounded by the cost and firm mix. This implies that as the level of real cost imposed on smuggling by enforcement approaches the export tax, the risk premium earned from smuggling is approaching zero. As the risk premium earned from smuggling declines, the number of firms smuggling declines. As firms switch to legal trade only, the domestic average wholesale price of exports approaches the full duty price ( $P^t$ ). Once the real cost of smuggling exceeds the export tax, all smuggling will be eliminated. Production and consumption will be at ( $P^t$ ,  $C^t$ ) and welfare ( $V^t$ ). The export tax distorted equilibrium will be the final result.

The introduction of parallel markets for exports and varying degrees of risk aversion among exporting firms bounds the welfare levels associated with the coexistence of smuggling and strictly legal trade. The welfare boundaries are: an upper bound of ( $p^{amp0}$ ,  $c^{amp0}$ ,  $v^{amp0}$ ) and a lower bound of ( $P^t$ ,  $C^t$ ,  $V^t$ ), as depicted in figure (1). The parallel market generates this result because the cost of smuggling to the firm represents a mix of a real



resource cost and penalties-confiscations. This cost mix is imposed on the smuggling firm by the government. Simply stated, for the smuggling firm with respect to smuggling profits, the cost mix is irrelevant to the firm's decision to smuggle.

It has been demonstrated that a real cost imposed on smuggling by government enforcement can not reduce welfare below the original export tax distorted welfare level. This approach extends the analysis of Martin and Panagariya, Bhagwati and Hansen, and Pitt. The welfare results of these authors could be duplicated in the parallel market model by assuming that at least a proportion of the firms in the export industry are risk seeking. This assumption produces the ambiguous welfare result arrived at in the earlier literature. Therefore, it is the overt or implied assumption of the exporting firm's attitude toward risk generating the ambiguous welfare results found in the earlier literature.<sup>20</sup>

### **The Welfare Effect of Government Enforcement**

The welfare analysis of smuggling in the economic literature has ignored the interaction between government enforcement and illegal transactions.<sup>21</sup> However, the Bhagwati and Hansen result of "the less smuggling the better" implies that government action should be taken against smugglers.

For the purpose of simplifying the analysis of this interaction, assume that government enforcement effort can be increased without any significant increase in cost. The government agency responsible for the enforcement of the export tax laws and reducing smuggling has three policy instruments at its disposal. First, it can increase enforcement activity to increase the probability of apprehending smugglers. Second, the government can increase the penalty for smuggling. Third, the government can lower the tax on exports. Based on the earlier results that smuggling reduces the negative welfare effect of the export tax, the welfare effect of changing each policy instrument is examined below.

The result of increasing the tax on exported goods in the parallel market model causes a decline in expected profits for smugglers and a decline in profits for firms involved in legal trade only. This counter intuitive effect is the result of a fixed international price for the small country's exports and the smuggling firm producing a joint product for export. Equation (7) demonstrates that for a fixed level of domestic production the

smuggling firm's successful smuggling profit level declines when the export tax is increased.

Equation (8) demonstrates, however, that the decline in ( $P^s$ ) will be less than the decline in ( $P^t$ ). This implies that the risk premium earned from smuggling, ( $\pi$ ), will increase and induce more firms to smuggle. Total exports will decline, but the smuggler's market share will increase. This is what Cooper calls "the secondary effect": "With competition from smuggling of any type, what appears to an indirect tax, the export tax, becomes a direct tax on the income of he who pays it, for he can not pass it on to his customers."<sup>22</sup> The result of ( $P^s$ ) declining by less than the increase in the export tax in the parallel market model is consistent with Cooper's empirical result of the domestic average wholesale price of imports increasing by less than the increase in the tariff.

Increasing government enforcement activity increases the probability of being caught in the illegal act of smuggling. As demonstrated in equations (14) and (17), increasing the probability of apprehension reduces expected profits and expected utility of illegal trade. This causes the number of firms involved in illegal trade to decline. Therefore, the welfare effect of an increase in the probability of apprehension is negative with respect to the results of the previous section.

If government enforcement is able to raise the probability of apprehension to one, illegal profits would equal ( $Y^s - F$ ) for firms involved in illegal activity. Firms would then base their

decision on whether to be involved in legal or illegal trade on a simple comparison between  $(Y^s - F)$  and  $(Y^l)$ . Government could then eliminate illegal trade by setting the monetary penalty high enough so that  $(Y^l) > (Y^s - F)$ , for all firms. On the other hand, if government set  $(\rho=0)$ , and if  $(Y^s > Y^l)$ , then all firms will smuggle. In this situation, assuming perfect competition and no real cost incurred in smuggling, the country would attain a free trade equilibrium.

An increase in the monetary penalty  $(F)$  for firms caught in illegal trade lowers the expected profits and expected utility of profits for firms involved in illegal trade, as demonstrated in equations (14) and (18). Therefore, the number of firms involved in illegal trade decline. Thus, the welfare effect of an increase in the monetary penalty is negative with respect to the earlier results of this essay.

Setting the monetary penalty at zero would eliminate legal trade and bring about a free trade equilibrium, if  $(Y^s > Y_l)$ . Setting the monetary penalty at infinity would eliminate all illegal trade. The export tax distorted equilibrium would be attained, if the probability of detection is non-zero.

The welfare results arising from a change in government enforcement in the parallel market model are in sharp contrast to the ambiguous welfare results found in the earlier literature. In the parallel market model, as the real resource cost increases from increased enforcement, risk averse firms begin to switch from illegal to legal trade. With this assumption incorporated

into the model, increased enforcement has an unambiguous negative welfare effect. Government not only affects expected profits from smuggling, but also imposes an increase in real cost on smuggling that is reflected in the smuggling function (G). Once the increase in real resource cost associated with smuggling becomes greater than the extra revenues earned from smuggling, all smuggling will end, if ( $U'' < 0$ ).

Another interesting issue concerning government policy and smuggling is which policy tool is most effective in reducing smuggling. Extending the research of Becker (1968), the effectiveness of ( $\rho$ ) and (F) in reducing the amount of smuggling depends on the attitude toward risk of the traded goods industry. Equations (34) and (35) below are elasticity measures for the expected utility of illegal profits with respect to the policy instruments ( $\rho$ ) and (F),

$$(34) \quad -\partial E(U)/\partial \rho \cdot \rho/U = \{U(Y^S) - U(Y^S - F)\} \cdot \rho/U,$$

$$(35) \quad -\partial E(U)/\partial F \cdot F/U = \rho \cdot U'(Y^S - F) \cdot F/U.$$

It is possible to use equations (34) and (35) to determine which government policy instrument is most effective in reducing smuggling. A direct comparison of equations (34) and (35), results in condition (36). It is now possible to perform an algebraic manipulation to arrive at condition (37). Condition (37) demonstrates that the individual relative effectiveness of the enforcement policy instruments is contingent on the attitude of the traded goods industry toward the risk associated with smuggling, ( $U'' > 0$ ,  $U'' = 0$ ,  $U'' < 0$ ),

$$(36) \{U(Y^s) - U(Y^s - F)\} \cdot \rho/U \begin{matrix} > \\ = \\ < \end{matrix} \rho \cdot U'(Y^s - F) \cdot F/U,$$

$$(37) \{U(Y^s) - U(Y^s - F)\} / F \begin{matrix} > \\ = \\ < \end{matrix} U'(Y^s - F).^{23}$$

Equation (37) reveals that a one percent increase in  $(\rho)$  generates a larger percentage reduction in smuggling than a one percent increase in  $(F)$ , if the traded goods industry has a preference for risk. This occurs because the decline in expected utility from smuggling profits is greater from the increase in  $(\rho)$ . If the traded goods industry is risk averse, then a one percent increase in  $(\rho)$  generates a smaller percentage reduction in smuggling than a one percent increase in  $(F)$ .

The above analysis has produced a very interesting result with regards to policy instrument choice. Assume that clandestine smuggling is carried out by non-risk averse criminal elements in a society. Assume the type of smuggling that uses under-invoicing, under-assessment and false declaration is carried out by risk averse legitimate firms. If increased use of policy enforcement instruments has a zero or equal cost, government could then choose the instrument which is most effective against the type of smuggling most prevalent in its economy.

One other comparison can be made. The elasticity measure for the expected utility of illegal profit, equation (38), with respect to the export tax is greater than the elasticities of the enforcement policy instruments,

$$(38) -\partial E(U)/\partial t \cdot t/U = \{-\rho \cdot U'(Y^s - F) \cdot dy/dP^s \cdot dP^s/dt - (1-\rho) \cdot U'(Y^s) \cdot dy/dP^s \cdot dP^s/dt\} \cdot t/U.$$

The comparison of equation (38) to equations (34) and (35) demonstrates that reducing the export tax by one percent generates a larger percentage reduction in smuggling than does a one percent increase in government enforcement. This result holds if it is assumed,  $(U'' \leq 0)$ .

Sheikh (1990) used a variant of Tobin's (1958) portfolio-selection risk model to investigate smuggling. The mean-variance methodology allowed Sheikh to reexamine the theoretical results of the smuggling literature, and he concludes that all existing models over predict the possible positive welfare effects of smuggling because the introduction of risk lowers welfare by its mere presence. Sheikh's model, however, does not provide an explanation for the coexistence of strictly legal firms with smuggling firms in the domestic market. He does, however, demonstrate that when smugglers are risk averse, increasing punishment is more effective in controlling smuggling than increases in the probability of detection. I arrive at the same result in equation (38) by using the Arrow-Pratt methodology to model the risk associated with smuggling.

The mean-variance approach has several weaknesses that were not addressed by Sheikh. The literature on "choice under uncertainty" reveals that generally the mean-variance approach will not be able to reproduce the "true" rankings of the complete distributions generated by the expected utility hypothesis approach.<sup>24</sup> The mean-variance approach, when used to model the risk associated with smuggling, will only produce suitable

approximations to the expected utility approach under one of the following conditions: 1) risks are small in some sense; 2) the distribution of smuggling profits must be normal; 3) the smuggling firm's utility function is quadratic. Sheikh's mean-variance model fails to make the necessary assumptions to meet any one of these three conditions.



### Summary

The purpose of this essay is to extend the analysis of Bhagwati and Hansen and other economists working on the welfare effect of illegal transactions. This essay's analysis began with the construction of the parallel market model for export smuggling. The results of the model are as follows: 1) the coexistence of legal and illegal trade in the domestic market for exports is possible when a domestic price differential exists; 2) smuggling is welfare enhancing, irrespective of the real resource cost associated with it, or smuggling will end; 3) increased government enforcement against smuggling has an unambiguously negative effect on welfare and smuggling market share; 4) the relative effectiveness of enforcement instruments is dependent on the risk preference of smugglers; and 5) the ambiguous welfare results attributed to smuggling in the earlier literature are the direct consequence of not addressing the risk issue associated with smuggling.

The basic conclusion of the essay for most situations is that smuggling is desirable over non-smuggling. The basic policy conclusion for lesser developed countries is clear. Lesser developed countries should not rush into a policy of eliminating smuggling without considering other courses of action. Otherwise, they may inflict a welfare loss upon their economies.

## Appendix (A)

The profit maximization first order condition for equation (5) is,

$$(1a) \quad \partial Y^l / \partial X = P^f \cdot (1-t) - C_x' = 0.$$

The profit maximization second order condition for equation (5) is,

$$(2a) \quad \partial^2 Y / \partial X^2 = -C_x'' < 0.$$

The second order condition for profit maximization holds for the legal trade only exporting firm under the assumption that the domestic export production function for the firm is strictly concave and therefore the firm has an upward sloping marginal cost curve, ( $C_x'' > 0$ ). Under the assumption that second order conditions given in equation (2a) hold, the legal trade only firm's profit function represented by equation (6) is well behaved.

### Appendix (B)

The profit maximization first order conditions for equation (7) are,

$$(1b) \quad \partial Y / \partial L = P^f \cdot G_L + P^f \cdot (1-t) - C_x' = 0,$$

$$(2b) \quad \partial Y / \partial S = P^f \cdot G_s - C_x' = 0.$$

The profit maximization second order conditions for equation (7) are,

$$(3b) \quad \partial^2 Y / \partial L^2 = P^f \cdot G_{LL} - C_x'' < 0,$$

$$(4b) \quad \partial^2 Y / \partial L \partial S = P^f \cdot G_{LS} - C_x'' < 0,$$

$$(5b) \quad \partial^2 Y / \partial S^2 = P^f \cdot G_{SS} - C_x'' < 0,$$

$$(6b) \quad \partial^2 Y / \partial S \partial L = P^f \cdot G_{SL} - C_x'' < 0.$$

The partial derivative ( $G_s$ ) is the marginal product of ex-ante smuggling in production of successful ex-post smuggling and is assumed to be positive. The partial derivative ( $G_L$ ) is the marginal product of legal trade in production of successful ex-post smuggling and is assumed to be positive. The partial derivative ( $C_x'$ ) is the marginal cost of domestic export production and is assumed to be positive. The second order partial derivatives ( $G_{SS}$ ) and ( $G_{LL}$ ) are assumed to be negative because of the strict concavity assumption imposed on ( $G$ ). The cross partial derivatives, ( $G_{SL}$ ,  $G_{LS}$ ), are assumed positive and small. This implies that the marginal productivity of either input increases if the other input is increased. The second order partial derivative ( $C_x''$ ) represents the slope of the firm's marginal cost curve for domestic export production and is assumed to be positive because of the strict concavity assumption imposed

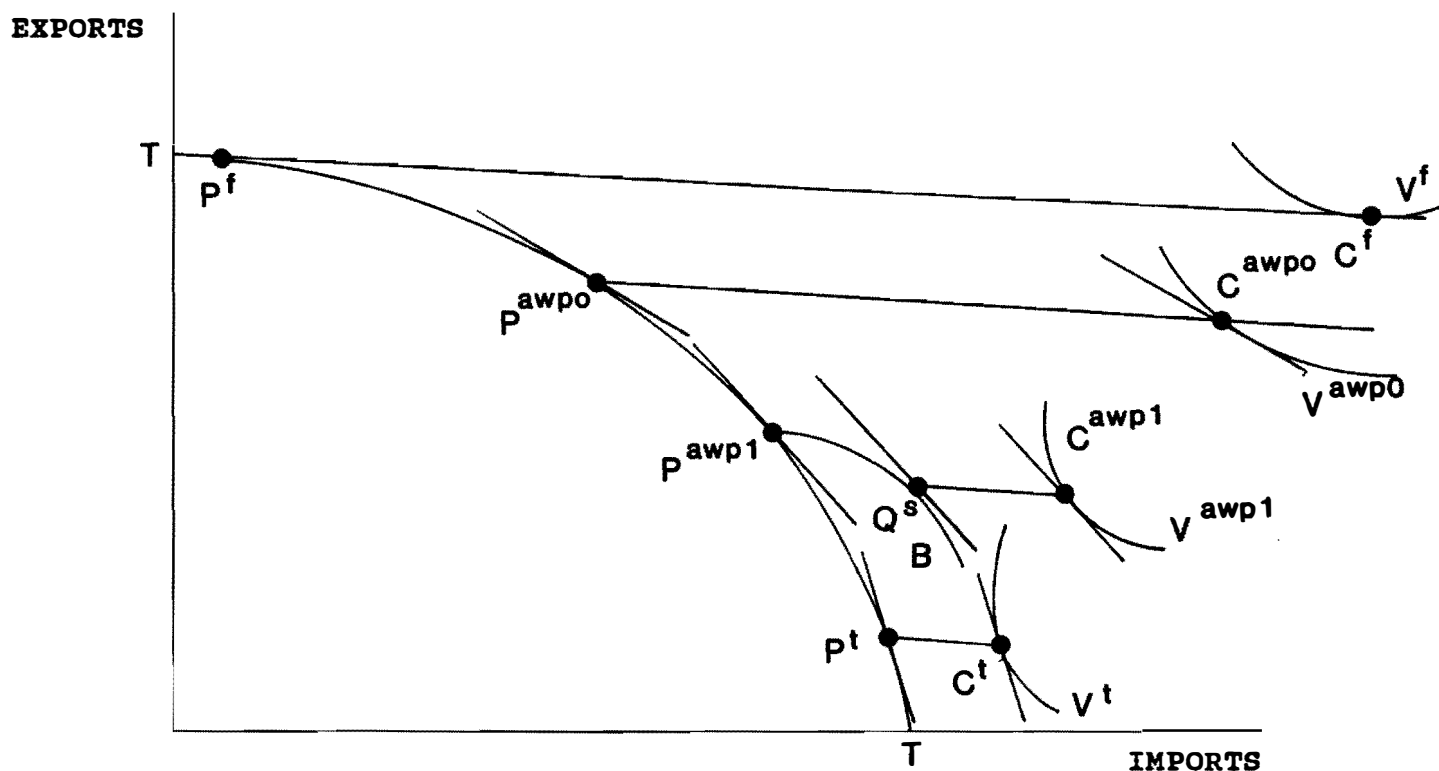
on the exporting firm's production function for the domestic production of exports. The second order conditions for profit maximization hold when it is assumed that the cross partial derivatives are positive and small, and it should be noted that it is assumed that changing (L) or (S) implies the production of exports (X) must increase.

$$(7b) \quad A = \begin{bmatrix} P^f \cdot G_{LL} - C_x'' & P^f \cdot G_{LS} - C_x'' \\ P^f \cdot G_{SL} - C_x'' & P^f \cdot G_{SS} - C_x'' \end{bmatrix}, \quad \text{DET } (A) > 0.$$

Under the assumption that second order conditions given in equation (7b) hold, the smuggling firm's profit function represented by equation (9) is well behaved.

## Appendix (C)

Figure (1)



The symbols  $(T, T)$  represent the production possibilities frontier for a small country. The variables  $(P^f, C^f, V^f)$  represent the free trade equilibrium position for the small country. The variables  $(P^t, C^t, V^t)$  represent the non-smuggling tax distorted equilibrium position for the small country. The variables  $(P^{awp0}, C^{awp0}, V^{awp0})$  represent the smuggling equilibrium position when smuggling does not incur a real resource cost over legal trade and strictly legal trade is coexisting with joint product illegal trade. The symbol  $(B)$  represents the inferior aggregate rate of transformation curve for smuggling when

smuggling incurs a real resource cost greater than legal trade. The variable  $(Q^s)$  represents the share of the export market that joint product smugglers control. Strictly legal trade will take equilibrium consumption to  $(C^{amp1})$ . Equilibrium domestic relative prices and welfare are  $(P^{amp1})$  and  $(V^{amp1})$  respectively. The welfare rankings for the different scenarios is clearly demonstrated in figure (1).

### References

- Becker, G., 1968, "Crime and Punishment: An Economic Approach", *Journal of Political Economy* 78, pp. 1-54.
- Bhagwati, J. and B. Hansen, 1973, "A Theoretical Analysis of Smuggling", *Quarterly Journal of Economics* 87, pp. 172-187.
- Bhagwati, J. and T.N. Srinivasan, 1973, "Smuggling and Trade Policy", *Journal of Public Economics* 2, pp. 377-389.
- Cooper, R., 1974, "Tariffs and Smuggling in Indonesia", in: J. Bhagwati, ed., "Illegal Transactions in International Trade", (North-Holland, Amsterdam), pp.183-192.
- Deardorff, A. and W.F. Stolper, 1988, "Effects of Smuggling under African Conditions: A Factual, Institutional and Analytic Discussion", Seminar Discussion Paper, No.230, University of Michigan.
- Deaton, A. and J. Muellbauer, 1987, "Economics and Consumer Behavior", (Cambridge University Press, London).
- Johnson, H.G., 1974, "Notes on the Economic Theory of Smuggling", in: J. Bhagwati, ed., "Illegal Transactions in International Trade", (North-Holland, Amsterdam), pp. 39-46.
- Kemp, M.C., 1976, "Smuggling and Optimal Commercial Policy", *Journal of Public Economics* 5, pp. 381-384.
- Lerner, A.P., 1936, "The Symmetry between Import and Export Taxes", *Economica* 3, pp. 306-313.
- Martin, L. and Panagariya, A., 1984, "Smuggling, Trade, and Price Disparity: A Crime Theoretic Approach", *Journal of International Economics* 17, pp. 201-217.
- Pitt, M., 1981, "Smuggling and Price Disparity", *Journal of International Economics* 11, pp. 447-458.
- Pratt, J., 1964, "Risk Aversion in the Small and in the Large", *Econometrica* 32, pp. 122-136.
- Samuelson, P., 1954, "The Transfer Problem and Transport Cost: Analysis of the Effects of Trade Impediments", *Economic Journal* 64, pp. 264-289.
- Scholer, K., 1989, "Risk and Illegal Trade", *Metroeconomica* 40, pp. 87-97.
- Sheikh, M.A., 1990, "A Theory of Risk, Smuggling and Welfare", *World Development* 17, pp. 1931-1944.

Tobin, J., 1958, "Liquidity Preference as Behavior Towards Risk", *Review of Economic Studies* 25, pp. 65-86.



### Endnotes

\*. I wish to thank Donald Coes, Robert Gillespie, Earl Grinols, Stan Herren, and the two anonymous referees for their comments. Any remaining errors are my responsibility.

1. An implicit assumption of the Bhagwati and Hansen model is that the real cost of smuggling is equal to the tariff.

2. See Bhagwati and Hansen (1973), p.172.

3. See Cooper (1974), pp.188-189.

4. Pitt (1981) provides empirical evidence of price disparity occurring in the Indonesian domestic market for commercial rubber.

5. Deardorff and Stolper (1988) and Cooper (1974) argue that smuggling may not impose a real cost on society.

6. The real resource cost may take the form of special packing costs necessary to hide smuggled goods. It could also come from shipping goods out of or into clandestine ports to avoid detection.

7. We appeal to the arguments presented against an excessive real resource cost associated with smuggling in the articles by Cooper (1974) and Deardorff and Stolper (1988) for this assumption.

8. See Cooper (1974), p.190.

9. The cost function is assumed to be twice differentiable, and cost minimization is assured by the assumption that the firm's production function for the domestic production of exports is strictly concave. The first and second order conditions for profit maximization are given in appendix (A).

10. The cost function for the firm is assumed to represent the minimum production cost associated with a specific level of output (X). The cost function embodies the assumption of strict concavity of the firm's production function and therefore produces (U) shaped average cost curves for the domestic export producing firm regardless of whether the firm smuggles or not. This assumption assures an interior maximum exists for the firm's profit function.

11. The first and second order conditions are given in appendix (B). Note that  $X = L+S$ .

12. See Cooper (1974), p.190.

13. It should be noted that as the expected value of (R) for the industry increases, the expected value of ( $\pi^*$ ) for the industry will also increase and therefore the number of firms smuggling must decline.

14. I shall assume that each exporting firm has equal weight in determining the domestic average wholesale price of exports.

15. Pitt (1981), p.450.

16. Pitt (1981), p. 454.

17. It should be noted that fines will be zero in the non-smuggling case.

18. The mathematical model developed here examines export smuggling; however, the geometry used in figure (1) applies equally to import and export smuggling in a manner analogous to the symmetry results demonstrated by Lerner (1936) for import and export taxes.

19. The real resource cost of smuggling is the excess cost incurred over legal trade and is due to the cloaking activities employed to hide illegal activity from government enforcement.

20. This point was made by Sheikh (1990).

21. The literature assumes government enforcement is constant. For example, see H.G.Johnson (1974) and Bhagwati and Srinivasan (1973).

22. Cooper (1974), page 190.

23. Becker (1968), used a similar approach in his analysis of the economics of crime., p.11.

24. For a discussion of this problem with the mean-variance approach, see Deaton and Muellbauer (1987), pp. 395-405.